

Chapter 3

Theory of elastic stress sensitivity

3.1 Stress and pressure: terminology and effective stress concept

The main intention of this section is to introduce the pressure and stress related terminology used in this thesis. It attempts to take the different definitions into account that can be found throughout the literature. This attempt is surely not be completed, but should help to place the ideas and results of this thesis more easily into the existing concepts. Beyond this, the second part of this section is dedicated to more general aspects of pressure terminology and gives a summary over some special units and unit conversion factors.

As a consequence of the broad interest in the role of pore pressure and the overall state of stress in the subsurface as well as the temporal evolution of these quantities from both, academics as well as industry, on the hand, the related research was pushed tremendously, but on the other hand confusion arose concerning the used terminology and units. In a recent publication Bruce & Bowers (2002) took care of clarifying pore pressure terminology. Apparently, one confusing aspect of pressure terminology arises from mixing the terms for absolute pressure and pressure gradients. Especially the latter might produce serious confusion since a gradient is always related to a reference point. In practice, this can be the seafloor, the water table or even a point on a rig. However, in the literature the terms "pressure" and "stress" are usually mixed when stress is treated as isostatic.

Basically, a stress is a tensorial quantity while a pressure is a scalar. In situations when all principal stress components are equal and shear stress is absent the state of stress is denoted as a pressure. Such a shear stress free state of stress in a rock is often called *hydrostatic*. Although this is common in rock physics literature, I will not follow this line. A shear stress free state of stress in a rock is denoted as *isostatic* and the term hydrostatic is restricted to fluid pressure (see Fig. 3.1). Distinguishing between both terms accounts for the fact that an isostatic stress in a rock results from the applied stress. In contrast, a hydrostatic pressure in a fluid results from a material property of fluids, namely, that they can not resist any shear force.

In the following, definitions are given for the terminology most frequently used in

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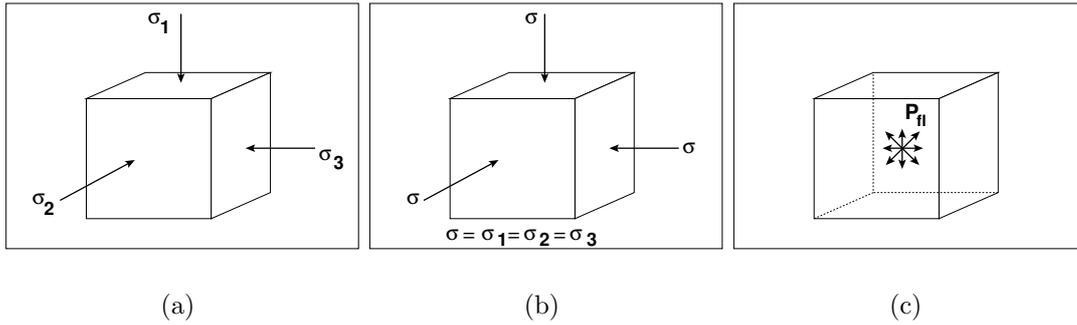


Figure 3.1: Fig. 3.1(a): Triaxial stress with $\sigma_1 \neq \sigma_2 \neq \sigma_3$, as usually found in the solid brittle Earth's crust. Fig. 3.1(b): Isostatic stress, as usually used in laboratory and frequently denoted as hydrostatic pressure. Fig. 3.1(c): True hydrostatic pressure P_{fl} as realized in fluids.

the literature. For instance, a *normal* or *hydrostatic* fluid pressure P_{fl} is defined as the pressure in a certain depth z that is in equilibrium with the pressure exerted by the weight of the overlying static fluid column with the fluid density ρ_{fl} , thus:

$$P_{fl}(z) = \int_{z=0}^z \rho_{fl}(z)gdz, \quad (3.1)$$

where g is the acceleration due to gravity.

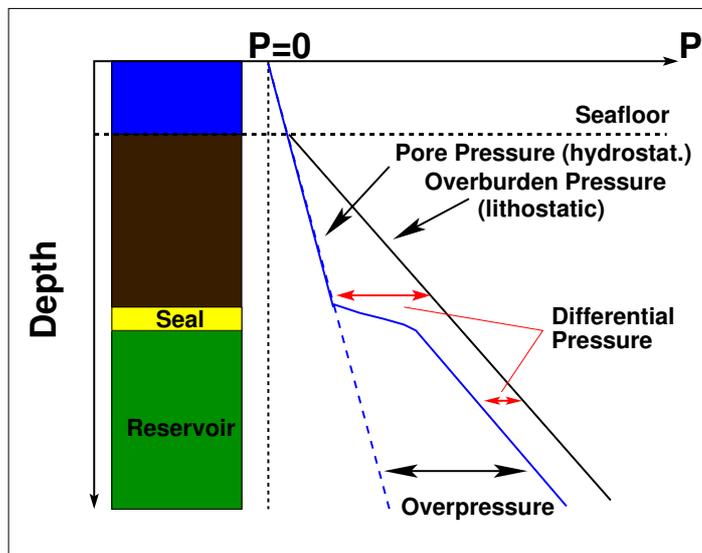


Figure 3.2: Sketch of typical pore, overburden and differential pressure profile. Above the seal the pore pressure increases linearly with a hydrostatic gradient with depth. Note, the usage of overburden and differential pressure instead of the correct terms overburden and differential stress accounts for the assumption of an isostatic state of stress.

The pressure exerted by the pore fluid is commonly called *pore pressure*. Equivalent expressions are *fluid pressure* or *formation pressure*. In many situations, especially in hydrocarbon reservoirs, the pore pressure is higher than normal pressure. The excess pressure above the normal pressure is called *overpressure* or *geopressure* (see Fig. 3.2).

In hydrocarbon reservoirs the pore fluid is a mixture of different fluid phases, e.g., brine, oil, and gas. They could show strong differences in specific density. The excess pressure created by the density differences between brine (or water) and hydrocarbons is called *Buoyant pressure*.

In the literature, (e.g., Dutta, 2002; Wang, 2001), the term *overburden pressure*, also called *confining pressure*, *lithostatic pressure*, or *geostatic pressure*, is used to express the stress at any depth which results from the weight of the overlying rock strata and the pore filling fluid (see Fig. 3.2). This notation implies that the considered state of stress is assumed to be isostatic. However, imagine a situation where tectonic stress, i.e., laterally acting external stress, is absent. At any depth the weight of the overburden induces a vertical stress and leads to compaction of the rocks. This situation is similar to a uniaxial compression test. In such a test, the rock shortens vertically and expands laterally. A measure for this deformation is the Poisson's ratio (see, eq.2.12). In the Earth's crust, vertically compressed rocks can not expand to the sides. Consequently, it can be shown from the theory of elasticity that the vertical stress σ_v , exerted by the weight of the overburden, induces a horizontal stress σ_{hor} (Thomsen, 1986):

$$\sigma_{\text{hor}} = \frac{\nu}{1 - \nu} \sigma_v. \quad (3.2)$$

For a commonly used Poisson's ratio $\nu = 0.25$, the induced horizontal stress is $1/3 \cdot \sigma_v$. Therefore, it is only consequent to talk about overburden (or confining) stress rather than overburden pressure. In contrast to field studies where the terms "overburden" and "confining" are both common, in laboratory experiments only the latter is used to denote the externally applied stress.

Below the water table the pore space can be assumed to be saturated by at least one fluid phase. Consequently, the actual state of stress in the subsurface results from both the confining stress and the pressure in the fluid. In other words, the overburden stress at any depth is carried by both, the rock matrix and the pore fluid. This obviously has consequences for the mechanics of fluid saturated porous rocks.

Since it is uncomfortable to formulate equations of the physics of pore mechanics in terms of confining stress σ^c and pore pressure P_{fl} it is common to combine them to a single *effective stress* σ^e . This is, what is usually called, the *concept of effective stress*. It was first formulated by Terzaghi (1936). However, his results derived empirically from experiments on different porous materials and theoretically were in contrast to each other. While the experiments stated that only the difference between confining stress and pore pressure was effective for rock and soil deformation, his theoretical considerations let him to the conclusion that the effective stress σ^e should equal $\sigma^c - \phi P_{\text{fl}}$, where ϕ is the porosity. Other theoretically derived results identify the Biot coefficient $\alpha = 1 - K_{\text{dry}}/K_0$ instead of the porosity to be multiplied with the pore pressure. Goulety (1998) discusses the effective stress for porosity reduction in shales in detail and reworks different compaction studies from literature. These studies state that in many geological situations the compaction of shales can not be fitted with the pure difference between vertical stress and pore pressure, but need an effective stress coefficient less than 1. Goulety states, that this need for an effective stress coefficient arises to compensate the error induced by using the wrong confining stress, namely, the vertical stress. In fact, the correct stress for compaction is the mean principal stress. Moreover, Berryman (1992) has shown that an effective stress coefficient has

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to be formulated for any rock property under investigation, i.e., there is no universal effective stress coefficient.

However, in general a stress or a pressure is called "effective" if it is formulated as the difference between confining stress (pressure) on the one side and the pore pressure times an effective stress coefficient n on the other side (eq. 3.3 and 3.5). In terms of pressure, hence, assuming an isostatic confining stress, we have per definition:

$$P_{\text{eff}} = P_c - n \cdot P_{\text{fl}} \quad (3.3)$$

$$P_{\text{diff}} = P_c - P_{\text{fl}}, \quad (3.4)$$

where n is an effective stress coefficient. The pure difference between confining pressure and pore pressure is denoted as *differential pressure* (eq. 3.4). A special case arises when $n = 1$ (eq. 3.4). In this case, the effective pressure equals the differential pressure. Sometimes, the difference between confining and pore pressure is also denoted as *net overburden pressure*. In terms of a confining stress these equations become

$$\sigma^e = \sigma^c + n \cdot \delta_{ij} P_{\text{fl}}, \quad (3.5)$$

$$\sigma^d = \sigma^c + \delta_{ij} P_{\text{fl}}, \quad (3.6)$$

where δ_{ij} is the Kronecker delta function. Following consequently the pressure naming scheme we might now denote σ^e as *effective* and σ^d as *differential* stress. Although this terminology is consistent with the described physics it leads to confusion with the definition for differential stress used in tectonophysics and rock mechanics. There, differential stress describes the difference between the minimum and maximum principal stress.

To avoid this conflict within this thesis "effective stress" is strictly used when the difference between confining stress σ^c and pore pressure P_{fl} is considered. If a special effective stress coefficient is needed this coefficient will be given explicitly. In fact, it will be shown later in section (3.2.1) that in many rocks the pure difference between confining stress and pore fluid pressure is effective for porosity changes.

The remarks made above should hopefully clarify the terminology used in this thesis. For understanding this thesis it is not necessary to read through the rest of the section. However, the last part of this section is dedicated to some further aspects of stress/pressure terminology and the problem of unit conversion.

Especially in the hydrocarbon industry pore pressure prediction is a highly interdisciplinary task. Geoscientists try to get assumptions about the subsurface stress and pore pressure. The resulting stress and pressure models are necessary for engineers to reduce risks while drilling and to optimize reservoir development and depletion. Unfortunately, the drilling community uses very different terminology related to mud weights instead of stresses or pressures.

Some of the more commonly used terms of the drilling community are (Dutta, 2002)

- *Differential pressure*: Drilling engineers use this term in a completely different sense than geoscientists. Here, it describes the difference between the hydrostatic pressure exerted by the mud weight and the formation fluid pressure.
- *Balanced drilling* denotes the case when the hydrostatic pressure of the mud column is in equilibrium with the formation fluid pressure, i.e., when the differential

pressure (as used by the drillers) is equal to zero. This situation happened only seldom.

- *Overbalanced drilling*: describes the case when the hydrostatic pressure of the mud column is greater than the formation fluid pressure. This is the usual case while drilling.
- *Underbalanced drilling*: In this case the formation fluid pressure exceeds the hydrostatic pressure of the mud column. If this happens hazardous blow outs can occur.
- *Effective circulating density*: describes the apparent increased mud density of a circulating mud column induced by an additional pressure ("backpressure") acting against the formation due to frictional effects in the mud column.

Drillers also prefer pressure or stress *gradients* rather than the absolute values. Doing this requires a clear definition of the reference point. A local pressure gradient defines pressure variations over small depth intervals. For a depth interval length approaching zero this gradient is equal to the slope of the pressure vs. depth curve. A simple example is the hydrostatic gradient of a water column due to the increasing weight of the column with depth. If the density of the water is constant with depth the hydrostatic gradient is constant.

A crucial gradient in reservoir management is the *fracture gradient*. It is a measure for the pressure gradient where fracturing is induced in the affected rock. A comparison between the fracture gradient of a reservoirs seal rock and the natural formation pressure gradient is important during reservoir exploration to get an assumption if the seal was fractured during its geological development and hydrocarbons might have escaped. Obviously, an artificially induced fracturing during reservoir development has to be avoided for the same reason.

Another important pressure gradient in practice is the *equivalent mud weight*. This is an example for difficulties that arise when different communities use different terminology. Here, talking about pressure gradients leads to a weight, because the mud weight is referred to a volume. This turns the weight into a density which converts to a gradient. Obviously, expressing a pressure in terms of a density is scientifically incorrect, but engineers use this for practical reasons. Equivalent mud weights are usually given in *lb/gallon*, pressure in *psi* and pressure gradients in *psi/foot*. The conversion factor between equivalent mud weight and pressure is $1 \text{ lb/gal} = 0.052 \text{ psi/ft}$ and mud density is related to pressure gradient as $1 \text{ psi/ft} = 2.31 \text{ g/cm}^3$. A table of the most frequently used conversion factors is given in Appendix (G)

3.2 Anisotropic stress sensitivity

3.2.1 Deformation of pore space

Up to section 3.2.5 a comprehensive overview about the theory of the stress sensitivity approach is given. A detailed description is given in Appendix (B) which reflects the paper of Shapiro & Kaselow (2003).

3.2. Anisotropic stress sensitivity

In order to quantify the deformation of the pore space geometry it is necessary to define quantitatively (a) the bulk and the pore space volume V_b and V_p , respectively, and (b) to introduce several compliances of the porous system which describe the deformation of the system due to an applied load. This will be done following and extending the approach of Brown & Korringa (1975).

The geometry of the sample bulk volume can be described in terms of a surface Σ covering the sample as shown in Fig. (3.3). The surface normal is defined positive in the outward direction. In the same way, a second surface Ψ is defined representing the inner surface of the rock, i.e., it covers the interconnected pore space. The permeability of the interconnected pore space is sufficient in order to equilibrate deformation induced pore pressure gradients within the sample. Per definition, the positive normal direction of the inner surface is pointing into the sample. Where the outer surface of the sample cuts a pore it coincides with the inner surface and simultaneously seals the pore. However, the normals point in opposite directions. In this way, it is possible to represent the

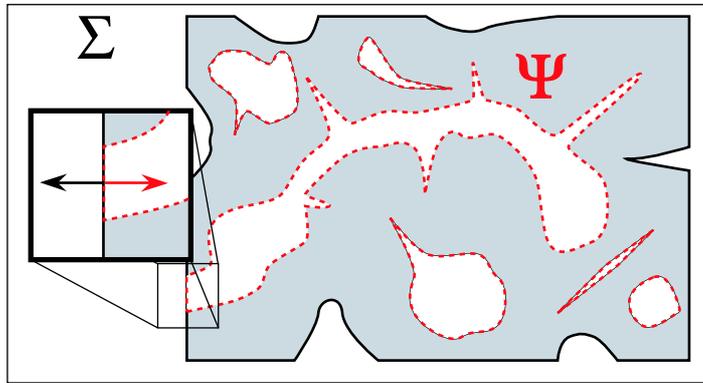


Figure 3.3: Sketch of bulk and pore space geometry. Both volumes are described in terms of covering surfaces Σ and Ψ , respectively. The positive normal direction of Σ points outward and in the case of Ψ into the sample. In 3D all pores are interconnected and effective for fluid flow.

bulk and pore space volume of the rock sample in terms of the encasing surfaces Σ and Ψ , respectively. Thus, it is possible to describe changes of both volumes by the displacement of points on the surfaces Σ and Ψ .

The rock may be subjected to two different load components, an externally applied confining stress σ_{ij}^c and an internally applied stress σ_{ij}^f , where i and j can be 1, 2, and 3. Here, the latter load component is denoted as pore stress. In most realistic situations this stress is a pressure, i.e., the pore pressure. Assume that the confining stress and/or pore stress have changed from an initial state of stress $(\sigma_{ij}^{c0}, \sigma_{ij}^{f0})$ to the current state $(\sigma_{ij}^c, \sigma_{ij}^f)$. As a result the bulk and pore space volume will be deformed. The deformation of both volumes can be described by the displacement of the corresponding surface points. All displacements are assumed to be very small in comparison to the size of the rock volume under consideration. This can be observed in many laboratory experiments where the deformations are usually in the order of 10^{-3} or even smaller.

The points of the external surface may have been displaced by $u_i(\hat{x})$, where \hat{x} is a surface point. Following Brown & Korringa (1975) it is possible to introduce a

symmetric tensor:

$$\eta_{ij} = \int_{\Sigma} \frac{1}{2}(u_i n_j + u_j n_i) d^2 \hat{x}. \quad (3.7)$$

Here, n is the surface normal at point \hat{x} . Since η_{ij} is related to the outer surface of the sample it will be denoted as the *outer surface tensor*. In the case of a continuous elastic body replacing the porous rock (i.e., a differentiable displacement is given at all its points) Gauss' theorem can be used to relate the integral over surface Σ to an integral over the corresponding volume.

$$\eta_{ij} = \int_{V_b} \frac{1}{2}(\partial_j u_i + \partial_i u_j) d^3 x. \quad (3.8)$$

The integrand here is the strain tensor and V_b is the sample bulk volume. Thus, $\epsilon_{ij} = \eta_{ij}/V_b$ is the volume averaged strain.

In this way the outer surface tensor η_{ij} can be related to the deformation of the rock sample. In the same way, it is possible to define a second symmetric tensor related to the deformation of the pore space.

$$\zeta_{ij} = \int_{\Psi} \frac{1}{2}(u_i n_j + u_j n_i) d^2 \hat{x}, \quad (3.9)$$

Here, \hat{x} is a point of surface Ψ , u_i is a component of the displacement of points \hat{x} , and n_i is a component of the outward normal of Ψ . In analogy to the outer surface tensor η_{ij} , the tensorial quantity ζ_{ij} will be denoted as the *inner surface tensor*.

If the pore space is completely filled with some material (e.g., a fluid or clay or cement) then, in analogy with the outer surface tensor η_{ij} , the pore volume averaged inner surface tensor ζ_{ij}/V_p will denote the volume averaged strain of this material, where V_p denotes the volume of the pore filling material. Moreover, using the summation convention, $-\zeta_{ii}$ denotes a volume change of the pore filling material.

Up to this point two tensorial quantities η_{ij} and ζ_{ij} were introduced which describe the bulk and pore space volume, respectively, in terms of encasing surfaces and the deformation of these volumes through the displacement of the corresponding surface points. However, these deformations result from the application of two stress fields. These stress fields will be explained in more detail in the following.

As common in rock mechanics, stress acting compressional with respect to the solid phase is defined negative. Assume that the stress σ_{ij}^f acting on the pore space surface Ψ is isostatic. In this case, the diagonal elements of the stress tensor are identical and shear stress is absent. This described the most realistic situation, namely, that the pore space is filled with a fluid. In the literature, this state of stress is, in general, denoted as hydrostatic, because it is typical for fluids. Since a more general situation is considered here, where the pore space can be filled with an arbitrary material, this state of stress is denoted as isostatic. In the case of an isostatic stress σ_{ij}^f it is possible to define a scalar pressure P_p acting on the pore space surface: $\sigma_{ij}^f = -\delta_{ij} P_p$.

As common in rock mechanics and already mentioned in section (3.1) both load components are usually combined. Here, the difference between the external confining stress and internal pore stress is defined as the effective stress σ^e :

$$\sigma_{ij}^e = \sigma_{ij}^c - \sigma_{ij}^f. \quad (3.10)$$

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In the case of an isostatic pore stress, this gives:

$$\sigma_{ij}^e = \sigma_{ij}^c + \delta_{ij}P_p. \quad (3.11)$$

Assuming further a complete isostatic state of stress, i.e., also the external confining stress is isostatic, gives:

$$\sigma_{ij}^e = \delta_{ij}\sigma_{ij}^c + \delta_{ij}P_p = (-P_c + P_p)\delta_{ij} = -(P_c - P_p)\delta_{ij}, \quad (3.12)$$

or, completely written in terms of pressure:

$$P_{\text{eff}} = P_c - P_p, \quad (3.13)$$

where P_{eff} and P_c are the effective and confining pressure, respectively.

After considering the load components acting on a porous rock rock compliances will be introduced which relate the acting stress to the deformation of the rock.

In analogy to the paper of Brown & Korrington (1975), three fundamental compliance tensors of an anisotropic porous body can be defined which can be obtained from appropriate laboratory measurement.

$$S_{ijkl}^{\text{dry}} = \frac{1}{V_b} \left(\frac{\partial \eta_{ij}}{\partial \sigma_{kl}^e} \right) \Bigg|_{\sigma^f}, \quad (3.14)$$

$$S_{ijkl}^{\text{mt}} = \frac{1}{V_b} \left(\frac{\partial \eta_{ij}}{\partial \sigma_{kl}^f} \right) \Bigg|_{\sigma^e}, \quad (3.15)$$

$$S_{ijkl}^{\text{p}} = -\frac{1}{V_p} \left(\frac{\partial \zeta_{ij}}{\partial \sigma_{kl}^f} \right) \Bigg|_{\sigma^e}. \quad (3.16)$$

Again, V_b is the bulk volume of the porous body and V_p is the volume of the interconnected pore space. S^{dry} denotes the compliance tensor of the drained rock matrix. It is obtained in an experiment where the rock samples strain (in the sense of eq. 3.7) is measured as a function of the effective stress while keeping σ_{ij}^f constant. This corresponds to a compressional experiment on a dry sample or a drained experiment where the confining stress is variable and the pore pressure is kept constant by letting the pore fluid freely entering or leaving the sample. S^{mt} and S^{p} are the compliance tensors characterizing the grain material and the pore space, respectively. They are obtained in experiments where the inner stress, i.e., the pore pressure, is changed, but the effective stress as defined above (eq. 3.11) is constant. Then, S_{ijkl}^{mt} and S_{ijkl}^{p} are obtained by relating the rock deformation to the bulk and pore volume, respectively.

The inner surface tensor ζ_{ij} and the bulk volume of the sample can be used to define an additional tensorial quantity, the so-called *generalized porosity*:

$$\phi_{ij} = \frac{\zeta_{ij}}{V_b}. \quad (3.17)$$

The generalized porosity is related to the porosity and can be used together with the above defined compliances of an anisotropic porous medium to derive an equation that formulates the stress dependence of porosity. For details on this derivation, see

Appendix (B). This relation for the stress dependence of porosity upon the effective stress and pore pressure reads:

$$\delta\phi = (S_{kl\bar{i}\bar{i}}^{\text{dry}} - S_{kl\bar{i}\bar{i}}^{\text{mt}} - \phi S_{mmkl}^{\text{dry}})\delta\sigma_{kl}^e - \phi(S_{i\bar{i}kk}^{\text{p}} - S_{mmkk}^{\text{mt}})\delta P_{\text{fl}}. \quad (3.18)$$

It provides an exact relation describing the dependence of porosity on changes in pore fluid pressure and confining stress. Hence, it formulates quite well the dependence of porosity on a geologically realistic state of stress, although is it limited to elastic, i.e, usually small, deformations. Consequently, it may provide a suitable formalism to estimate porosity reduction with burial depth for already consolidated rocks in real geological environments. In practice, the in situ stress is usually approximated with a confining pressure, identical to the overburden pressure. Thus, taking also an isostatic confining stress into account (i.e., also $\sigma_{kl}^e = -\delta_{kl}P_{\text{eff}}$) eq. (B.23) reduces to:

$$\delta\phi = (S_{ll\bar{i}\bar{i}}^{\text{dry}} - S_{ll\bar{i}\bar{i}}^{\text{mt}} - \phi S_{mml\bar{i}\bar{i}}^{\text{dry}})\delta P_{\text{eff}} - \phi(S_{i\bar{i}kk}^{\text{p}} - S_{mmkk}^{\text{mt}})\delta P_{\text{fl}}. \quad (3.19)$$

Equation (3.19) shows that generally the porosity is a function of both, the effective stress as well as the pore pressure. Remember, effective stress is defined as the pure difference between confining stress and pore pressure. If $S_{i\bar{i}kk}^{\text{mt}} = S_{i\bar{i}kk}^{\text{p}}$, i.e., the rock matrix is homogenous and/or the interconnected porosity is small then porosity changes depend on the difference between confining stress and pore pressure only.

The stress sensitivity approach provides several precise results describing stress dependencies of the pore space geometry. The compliance tensors S^{mt} and S^{p} are practically independent of effective stress at least up to a few hundred MPa. Thus, in equation (3.19) only two quantities are significantly stress dependent: the porosity ϕ and the dry rock matrix compliance tensor S^{dry} . Since porosity variations depend on the dry rock compliances and the dry rock compliances depend, in turn, on the porosity at least one more equation is required, which would mutually relate them. This equation cannot be obtained exactly, since S^{dry} depends upon the complete geometry of the pore space rather than on the magnitude of the porosity alone. Thus, a further analysis requires to involve some empirical observations and heuristic assumptions.

3.2.2 Elastic compliances

Typical stress dependencies of elastic moduli, hence, seismic velocities, look like shown in Fig. (3.4). Increasing effective stress leads first to a rapid non-linear increase of seismic velocities. Then, for higher stresses, the velocity stress dependence tapers into a flat linear relation. Occasionally, the linear part of the velocity-stress relation does not show any further increase of velocities with increasing stress, at least up to some hundred MPa (approx. 200-400 MPa, depending on the rock) effective stress. Although it is a quite intuitive assumption that this velocity dependence upon stress results from the closure of the porosity, many porosity measurements show that porosity does not change at all or even very slightly while velocities change remarkably (e.g., Khaksar *et al.*, 1999). Thus, it is a common interpretation that the rapid increase of velocities at low stresses results from the closure of cracks and grain contact vicinities. This part of the porosity, denoted as the *compliant porosity* usually represents only a very small fraction of the total porosity (< 1% in typical sandstones). Hence, even its complete

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closure does not change the bulk porosity remarkably. When this easily deformable part of the bulk porosity is closed the velocity increase is caused by the closure of the hardly deformable remaining stiff pores.

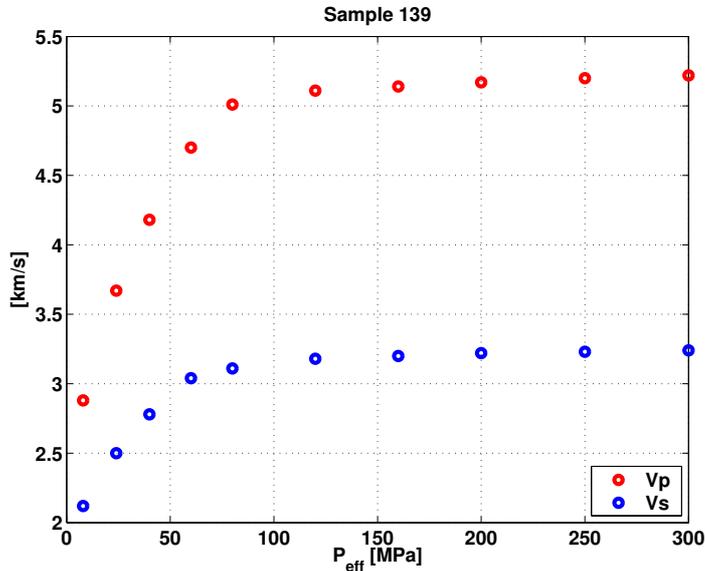


Figure 3.4: Typical ultrasonic P- (red circles) and S-wave (blue circles) velocity as a function of isostatic effective stress. This example shows a sandstone from SALZWEDEL drilling site (data from Freund, 1992).

This distinct deformation behavior of stiff and compliant porosity is taken in account by formulating:

$$\phi = \phi_c + [\phi_{s0} + \phi_s], \quad (3.20)$$

as already done by Shapiro (2003). Here, ϕ is the bulk interconnected porosity, ϕ_c is the compliant porosity supported by cracks and grain contact vicinities. As a rule of thumb, compliant porosity shows an aspect ratio γ (a relationship between the minimum and maximum dimensions of a pore) less than 0.01 (see Zimmerman *et al.*, 1986). The second part, $[\phi^{s0} + \phi^s]$, comprises the stiff porosity supported by more or less isometric or spherical pores (i.e., equidimensional or equant pores, see also Hudson *et al.*, 2001; Thomsen, 1995). The aspect ratio of such pores is typically larger than 0.1. Such a separation of the porosity into a compliant and a stiff part is very similar to the definitions of stiff and soft porosity by Mavko & Jizba (1991) and others.

In turn, stiff porosity is further separated into a stress independent part ϕ_{s0} , which is equal to the stiff porosity in the case of an effective stress $\sigma^e = 0$, and into a part ϕ_s which describes an amount of stiff porosity due to a deviation of the effective stress from zero. As mentioned above it is reasonable to assume that the relative changes of the stiff porosity, ϕ_s/ϕ_{s0} , are small. In contrast, the relative changes of the compliant porosity $(\phi_c - \phi_{c0})/\phi_{c0}$ can be very large, i.e., of the order of 1 where ϕ_{c0} denotes the compliant porosity in the unloaded case $\sigma^e = 0$. In general, the compliant porosity is a very small quantity since it represents only a very small part of the bulk porosity. As a rule of thumb, the amount of compliant porosity is much smaller than the stress independent part of the stiff porosity ϕ^{s0} and even than the absolute value of the stress induced change of stiff porosity ϕ^s . Thus, the following inequality is usually valid:

$$\phi_{s0} \gg |\phi_s| \gg \phi_c. \quad (3.21)$$

In sedimentary rocks the orders of magnitude for these quantities are approximately $\phi_{s0} = 0.1$, $|\phi_s| = 0.01$ and $\phi_c = 0.001$. Inequality (3.21) may not be valid in low porosity crystalline rocks. For instance, calculating the stress dependence of crack porosity from strain measurements, as introduced by Brace (1965), Kern *et al.* (1991) implicitly assume that the porosity of the KTB rocks consist of compliant porosity only. However, this has no implication on the following considerations since even if a rock shows no stiff porosity eq. (3.20) is still valid.

The definitions above are somehow "asymmetric" definitions of stiff and compliant porosity. This means, the stiff porosity is separated into a stress dependent and a stress dependent part while the crack porosity is treated as a whole. This is justified by their distinct deformation behavior and accounts for the assumption that under moderate loads considered here (approx. 200 - 300 MPa, dependent on the rock under consideration) the stiff porosity suffers small changes only. In contrast to this, the compliant porosity can be significantly changed or even completely closed. Thus, the notation system introduced above is convenient for describing such an "asymmetric" behavior.

Taking into account that both stress dependent parts ϕ^s and ϕ^c of the porosity introduced above are very small (of the order of the strain), it is reasonable to assume a first, linear approximations of the skeleton compliances as functions of these quantities. Using the equivalent stress dependent parts of the generalized porosity a Taylor expansion gives:

$$S_{ijkl}^{\text{dry}}(\phi_{mn}^{s0} + \phi_{mn}^s, \phi_{mn}^c) = S_{ijkl}^{\text{drys}} + C^{\text{drys}}\theta_{ijklmn}^s\phi_{mn}^s + C^{\text{drys}}\theta_{ijklmn}^c\phi_{mn}^c, \quad (3.22)$$

where S_{ijkl}^{drys} is the drained compliance tensor of a hypothetical rock with a closed compliant porosity (i.e., $\phi_c = 0$) and the stiff porosity equal to ϕ_{s0} . Further,

$$\theta_{ijklmn}^s = \frac{1}{C^{\text{drys}}} \frac{\partial S_{ijkl}^{\text{dry}}}{\partial \phi_{mn}^s}, \quad (3.23)$$

$$\theta_{ijklmn}^c = \frac{1}{C^{\text{drys}}} \frac{\partial S_{ijkl}^{\text{dry}}}{\partial \phi_{mn}^c}, \quad (3.24)$$

where the derivatives are taken in points $\phi_s = 0$ and $\phi_c = 0$, respectively. The θ quantities in eq. (3.23) and (3.24) are individual for a given rock sample. We assume that same load paths will give the same configuration of ϕ_{ij}^s and ϕ_{ij}^c . This means, only non-hysteresis deformations are considered. As shown in Appendix (B.2) the tensorial quantity θ_{ijklmn}^c is the most important rock characteristic for the stress dependence of compliances and seismic velocities of porous rocks. This tensor is denoted as the *Tensor of stress sensitivity*. It is analogous to the scalar dimensionless quantity introduced by Shapiro (2003) as the piezosensitivity for isotropic rocks under isostatic load.

The symmetry of this tensor reflects to the symmetry of the drained matrix compliance tensor of the rock under consideration. For example, the stress sensitivity tensor of a triclinic medium has 56 independent components and 3 in the case of an isotropic medium (see Appendix B.7 for details). The complexity of the stress sensitivity tensor reflects the highly complex variety of possible reactions of elastic moduli of porous systems due to applied stress.

3.2.3 Stress dependence of porosity

Assume that stress induced changes of stiff and compliant porosity are independent of each other. In this case, if the compliant pore space is closed then $\phi_{ij}^c = 0$. Then, as shown by eq. (B.36) in Appendix (B.3), the stress dependence of the stiff part of the generalized porosity reads:

$$-\delta\phi_{ij}^s = (S_{klij}^{\text{drys}} - S_{klij}^{\text{mt}})\delta\sigma_{kl}^e. \quad (3.25)$$

If this is valid then this relationship will also be valid for an arbitrary and usually small ϕ_c . Therefore,

$$-\delta\phi_{ij}^c = C^{\text{drys}}\theta_{klijmn}^c\phi_{mn}^c\delta\sigma_{kl}^e. \quad (3.26)$$

Taking into account that $\phi_s = 0$ if no load is applied yields:

$$\phi_{ij}^s = (-S_{klij}^{\text{drys}} + S_{klij}^{\text{mt}})\sigma_{kl}^e. \quad (3.27)$$

In order to derive an equation for the stress dependence of the elements of the generalized compliant porosity tensor ϕ_{ij}^c similar to eq. (3.27) simplifying assumptions have to be made. The derivation of this equation is given in Appendix (B.3). Considering orthorhombic media and assuming that the principal axes of the effective stress tensor are aligned within the symmetry axes of the medium it is possible to show that the stress dependencies of the diagonal elements ϕ_{ii}^c are given as:

$$\phi_{11}^c = \phi_{11}^{c0} \exp(-\theta_1^c \sigma_{11}^e C^{\text{drys}}), \quad (3.28)$$

$$\phi_{22}^c = \phi_{22}^{c0} \exp(-\theta_2^c \sigma_{22}^e C^{\text{drys}}), \quad (3.29)$$

$$\phi_{33}^c = \phi_{33}^{c0} \exp(-\theta_3^c \sigma_{33}^e C^{\text{drys}}). \quad (3.30)$$

where a new notation θ_1^c , θ_2^c and θ_3^c for θ_{111111}^c , θ_{222222}^c and θ_{333333}^c , respectively, was introduced.

The basic assumption for the above mentioned considerations is that the dry rock compliance tensor and the pore space are the rock characteristics most sensitive to changes in effective stress. A mutual relation between both quantities was found by taking the distinct deformation behavior of the stiff and compliant porosity into account. Especially the stress induced closure of compliant porosity causes significant stress induced changes of the dry rock compliance tensor. The new introduced rock characteristic which describes the changes of the dry rock compliance tensor with changes of generalized compliant porosity is the tensor of stress sensitivity. Taking the most general case of an arbitrary anisotropic medium under non-isostatic load into account shows that this tensor is of rank six. It is directly related to non-linear elasticity of porous rocks. Detailed considerations about the tensor of stress sensitivity and the deformation of the pore space are limited to orthorhombic media (for details see Appendix B.3). The principal components of the effective stress tensor are assumed to be aligned within the symmetry planes of the medium. These considerations include media that represent special cases of orthorhombic symmetry, i.e., TI, cubic, and isotropic media. The stress sensitivity tensor of an isotropic medium has only three independent entries. As shown in Appendix (B.3) the only realistic situation relevant for a rock physical application arises if two of these entries are identically zero and only one independent non-zero entry remains. In this case, the stress dependencies of the principal elements of the generalized compliant porosity tensor are independent from each other.

3.2.4 Stress dependence of elastic moduli

Although numerous simplifying assumptions were made the derived stress dependencies of the dry rock compliances and the (generalized) porosity are still quite general with respect to rocks and the state of stress that can be expected in rock physical practice. Thus, in the following an arbitrary elastic characteristic Λ (e.g., a seismic velocity, a stiffness or a compliance) of a porous drained body is considered.

However, this requires to take only those elastic characteristics into account that dependent on stress via the stress dependence of the pore space. Thus, let us assume that Λ , in a vicinity of the state where ϕ_{s0} and ϕ_c as well as the effective stress are identically zero, can be expanded into a Taylor series similar to eq. (3.22) with respect to the porosity (this should be valid for all such characteristics like seismic velocities and elastic moduli):

$$\Lambda(\phi_{ij}^{s0} + \phi_{ij}^s, \phi_{ij}^c) = \Lambda^{drys} [1 + \theta_{ij}^{s\Lambda} \phi_{ij}^s + \theta_{ij}^{c\Lambda} \phi_{ij}^c], \quad (3.31)$$

where only the linear part of the Taylor expansion were kept. Λ^{drys} is a hypothetical rock characteristic, since it assumes that ϕ_{s0} and P_{eff} equal zero and ϕ_c is zero as well. This is an unrealistic situation for rocks since it implicitly means that the rock would have no crack-like porosity at all. Hence, the mentioned formulation of the Taylor expansion is purely a mathematical concept. Here,

$$\theta_{ij}^{s\Lambda} = \frac{1}{\Lambda^{drys}} \frac{\partial \Lambda}{\partial \phi_{ij}^s}, \quad (3.32)$$

$$\theta_{ij}^{c\Lambda} = \frac{1}{\Lambda^{drys}} \frac{\partial \Lambda}{\partial \phi_{ij}^c}, \quad (3.33)$$

and the derivatives are taken at $\phi^s = 0$ and $\phi^c = 0$, respectively. Substituting the corresponding stress dependencies of the generalized porosity (equations 3.27 and 3.28 - 3.30) into eq. (3.31) gives:

$$\Lambda(\tau) = \Lambda^{drys} \left[1 + \theta_{ij}^{s\Lambda} \left(S_{ijK}^{mt} - S_{ijK}^{drys} \right) \sigma_K^e + \theta_I^{c\Lambda} \phi_I^{c0} \exp(-\theta_I^c C^{drys} \sigma_I^e) \right], \quad (3.34)$$

where K and I can assume one of values 1, 2 or 3 denoting 11, 22 and 33, respectively. In the exponent there is no summation over repeating indices.

A comparison of eq. (3.34) results with equation (1.1) shows a surprising but striking result. In the case of isostatic load and if the tensor of stress sensitivity of the rock corresponds to one of an isotropic medium all mentioned stress dependencies have the same form $A + KP - B \exp(-PD)$. Thus, the theoretically derived stress dependencies of all mentioned elastic rock characteristics have the same form as the empirically found best fit equation. Especially the physical meaning of parameter D is important. If the medium is isotropic or the stress sensitivity tensor corresponds to one of an isotropic medium the fit parameter D reads

$$D = \theta^c C^{drys}. \quad (3.35)$$

Thus, it is independent from the property under consideration, in other words, it is a universal quantity for the mentioned rock characteristics.

3.2. Anisotropic stress sensitivity

If Λ is understood in terms of seismic velocities this a rough approximation since the use of a Taylor expansion implies small changes of velocities with stress. In the case of significant velocity variations due to an applied stress this approximation might become erroneous. However, it was found that the error introduced by using an equation of the form $A + KP - B \exp(-PD)$ for seismic velocities is usually negligible, as will be shown in section (3.3).

3.2.5 Stress dependence of elastic anisotropy

Since a quite general relation of the stress dependent dry rock compliances has already been derived it is only consequent to analyze the corresponding stress dependence of elastic anisotropy. Since significant changes of elastic rock characteristics are produced by changes of compliant porosities only it is reasonable to neglect the contributions of the stiff porosity to stress dependences of elastic moduli in the following. Remember that orthorhombic symmetry is considered. However, the limitation to situations where the principal axes of the effective stress tensor are oriented normal to the symmetry planes is strongly restrictive and may only be satisfied in carefully conducted laboratory experiments.

Here, only weak anisotropic media are considered. In this case, the dry rock compliance tensor S_{ijkl}^{drs} of the medium is weakly anisotropic only or may even be approximated as isotropic. In addition, the stress sensitivity tensor and ϕ_{ij}^{c0} are assumed to show an even weaker anisotropy. This means that the stress sensitivity tensor corresponds effectively to one of an isotropic medium. As shown in Appendix (B.5) Tsvankin's parameters simplify to:

$$\epsilon^{(1)} = \epsilon_0^{(1)} + A_1 \theta^c \phi^{c0} (E_2 - E_3), \quad (3.36)$$

$$\epsilon^{(2)} = \epsilon_0^{(2)} + A_1 \theta^c \phi^{c0} (E_1 - E_3), \quad (3.37)$$

$$\delta^{(1)} = \delta_0^{(1)} + A_1 \theta^c \phi^{c0} (E_2 - E_3), \quad (3.38)$$

$$\delta^{(2)} = \delta_0^{(2)} + A_1 \theta^c \phi^{c0} (E_1 - E_3), \quad (3.39)$$

$$\delta^{(3)} = \delta_0^{(3)} + A_1 \theta^c \phi^{c0} (E_2 - E_1), \quad (3.40)$$

$$\gamma^{(1)} = \gamma_0^1 + A_2 \theta^c \phi^{c0} (E_3 - E_2), \quad (3.41)$$

$$\gamma^{(2)} = \gamma_0^2 + A_2 \theta^c \phi^{c0} (E_3 - E_1), \quad (3.42)$$

where ϵ_0 , δ_0 , and γ_0 refer to these parameters in the unstressed state. Here,

$$A_1 = \frac{2}{3} \frac{S_{11}}{S_{44}(S_{44} - 4S_{11})}, \quad (3.43)$$

$$A_2 = \frac{1}{6} \frac{1}{S_{44}}, \quad (3.44)$$

$$E_i = \exp(-\theta_i^c \sigma_{ii}^e C^{dry}), \quad (3.45)$$

where no summation over repeated indices is applied in eq. (3.45). If stress changes are small the exponential functions E_i in these equations can be expanded in Taylor series. In the special case of a completely isostatic load and an isotropic tensor of stress sensitivity all E_i terms become equal, i.e.,

$$E_1 = E_2 = E_3 = E = \exp(-\theta_c C^{dry} P_{\text{eff}}) \quad (3.46)$$

Hence, in equations (3.36) to (3.42) the right hand sides reduce to the initial values of the anisotropy parameters in the unstressed state. This means, if the rock is weakly anisotropic and the stress sensitivity corresponds effectively to one of an isotropic medium the initial elastic anisotropy of the rock will not change under an isostatic stress.

Considering initially isotropic rocks the situation reduces to the one considered in Shapiro (2003). Quantity θ^c is then the piezosensitivity introduced in that paper. Moreover, even in the case of initially anisotropic rocks only one single quantity $D = \theta_c C^{dry}$ controls the exponential parts of the stress dependence of any compliance, of any stiffness and of any elastic wave velocity. It will be shown in section (4.3) that there are anisotropic sedimentary and as well as metamorphic rocks which show this interesting phenomenon.

3.3 Isotropic stress sensitivity

In the previous sections a general first order stress dependence of anisotropic elastic rock characteristics under non-isostatic effective stress was derived. Although almost all rocks or geological formations show at least weak anisotropy and the state of stress in the Earth is always non-isostatic the simple assumption of isotropic rocks and isostatic stress is the most frequently and successfully used one in seismics and rock physics. Thus, in the following, the stress dependence of elastic moduli and seismic velocities in isotropic rocks under isostatic load will be considered. The constitutive equations were first introduced by Shapiro (2003). There, the stress dependencies of the dry matrix bulk and shear moduli were derived in the same way as shown for the more general case of anisotropic rocks under non-isostatic load.

As given by eq. (B.27) the compliances of a rock are related to the dry matrix bulk modulus, grain bulk modulus, and pore bulk modulus, K_{dry} , K_0 , and K_p , respectively, according to:

$$\frac{1}{K_{dry,mt,p}} = S_{1111}^{mt,dry,p} + S_{2222}^{mt,dry,p} + S_{3333}^{mt,dry,p} + 2(S_{1122}^{mt,dry,p} + S_{1133}^{mt,dry,p} + S_{2233}^{mt,dry,p}) \equiv S_{iikk}^{mt,dry,p}.$$

Note, in the literature (e.g., Mavko *et al.*, 1998) bulk moduli K_{mt} and K_p are usually denoted as K_0 and K_ϕ , respectively. In the following, the latter notation is used.

Replacing the compliances in eq. (3.19) with the corresponding bulk moduli as given above leads to stress dependence of porosity in isotropic rocks under isostatic load in terms of bulk moduli:

$$\delta\phi = \left(\frac{1}{K_{dry}} - \frac{1}{K_0} - \phi \frac{1}{K_{dry}} \right) \delta P_{eff} - \phi \left(\frac{1}{K_\phi} - \frac{1}{K_0} \right) \delta P_{fl}. \quad (3.47)$$

In analogy to the corresponding equation for anisotropic rocks (eq.B.22) eq. (3.47) shows that the pure difference between isostatic confining stress and pore pressure is effective for stress induced porosity variations if the porosity is small and/or the rock is in the Gassmann limit (i.e., $K_0 = K_\phi$). As done in the case of anisotropic rocks, in the following, it is assumed that the rock is in the Gassmann limit.

3.3. Isotropic stress sensitivity

Again, assuming that the dry matrix bulk modulus can be approximated as a function of stiff and compliant porosity in terms of a Taylor expansion at $K_{\text{dry}} = K_{\text{dryS}}$ as shown for the dry rock compliances, gives:

$$\frac{1}{K_{\text{dry}}}(\phi_{s0} + \phi_s, \phi_c) = \frac{1}{K_{\text{dryS}}} [1 + \theta_s \phi_s + \theta_c \phi_c], \quad (3.48)$$

with

$$\theta_s = K_{\text{dryS}} \frac{\partial C_{\text{dry}}}{\partial \phi_s}, \quad (3.49)$$

$$\theta_c = K_{\text{dryS}} \frac{\partial C_{\text{dry}}}{\partial \phi_c}. \quad (3.50)$$

K_{dryS} is a hypothetical bulk modulus of the rock in the unstressed state if the porosity of the rock would consist of the stress independent part of the stiff porosity only. Shapiro (2003) introduced θ_c as a new rock physical characteristic, the elastic piezosensitivity. This notation was changed to stress sensitivity to avoid confusion arising from misleading associations with effects of piezoelectricity. Remember, the scalar stress sensitivity represents the single effectively independent entry of the stress sensitivity tensor in the case of an initially isotropic medium (for details, see Appendix B.7).

Using equation (3.48) with the stress dependent formulations for ϕ_s and ϕ_c as given by Shapiro (2003) yields for the stress dependence of the dry matrix bulk modulus:

$$K_{\text{dry}}(P_{\text{eff}}) = K_{\text{dryS}} \left[1 + \theta_s \left(\frac{1}{K_{\text{dryS}}} - \frac{1}{K_0} \right) P_{\text{eff}} - \phi_{c0} \theta_c \exp \left(-\frac{\theta_c}{K_{\text{dryS}}} P_{\text{eff}} \right) \right]. \quad (3.51)$$

In the same way a corresponding equation for the dry matrix shear modulus can be derived which reads

$$\mu_{\text{dry}}(P_{\text{eff}}) = \mu_{\text{dryS}} \left[1 + \theta_{s\mu} \left(\frac{1}{K_{\text{dryS}}} - \frac{1}{K_0} \right) P_{\text{eff}} - \phi_{c0} \theta_{c\mu} \exp \left(-\frac{\theta_c}{K_{\text{dryS}}} P_{\text{eff}} \right) \right]. \quad (3.52)$$

As expected from the stress dependence of dry compliances, the stress dependencies of K_{dry} and μ_{dry} also have the form of eq. (1.1). Moreover, Shapiro (2003) has shown that the stress dependence of the saturated bulk modulus corresponds to eq. (1.1) as well.

Once the stress dependencies of K_{dryS} and μ_{dryS} are derived it is straightforward to obtain the stress dependencies of P- and S-wave velocities. Therefore, in the case of dry rock velocities the bulk and shear moduli given in eq. (2.14) and (2.13) are replaced by the corresponding stress dependent relations (3.51) and (3.52). Hereby, it is reasonable to neglect the stress dependence of the rock density. Thus, the stress dependence of seismic velocities of isotropic rocks under isostatic effective stress in terms of the stress

sensitivity approach read:

$$V_{\text{Sdry}}(P_{\text{eff}}) = \left[\left[\mu_{\text{dryS}} \left[1 + \theta_{s\mu} \left(\frac{1}{K_{\text{dryS}}} - \frac{1}{K_0} \right) P_{\text{eff}} - \phi_{c0} \theta_{c\mu} \exp \left(-\frac{\theta_c}{K_{\text{dryS}}} P_{\text{eff}} \right) \right] \right] \rho^{-1} \right]^{1/2} \quad (3.53)$$

$$V_{\text{Pdry}}(P_{\text{eff}}) = \left[\left[K_{\text{dryS}} \left[1 + \frac{4}{3} \mu_{\text{dryS}} + \left(\theta_s + \frac{4}{3} \theta_{s\mu} \right) \left(\frac{1}{K_{\text{dryS}}} - \frac{1}{K_0} \right) P_{\text{eff}} - \left(\theta_c + \frac{4}{3} \theta_{c\mu} \right) \phi_{c0} \exp \left(-\frac{\theta_c}{K_{\text{dryS}}} P_{\text{eff}} \right) \right] \right] \rho^{-1} \right]^{1/2} \quad (3.54)$$

Obviously, the stress dependencies of seismic velocities have the form

$$V(P) = \sqrt{A + KP - B \exp(-DP)}. \quad (3.55)$$

Shapiro (2003) gives linear approximations of eq. (3.53) and (3.54) assuming small variations (approx. 10% or smaller) of the velocities with stress. These approximations read:

$$V_{\text{Sdry}}(P_{\text{eff}}) \approx V_{\text{SdryS}} + \frac{1}{2} V_{\text{SdryS}} \theta_{s\mu} \left(\frac{1}{K_{\text{dryS}}} - \frac{1}{K_0} \right) P_{\text{eff}} - \frac{1}{2} V_{\text{SdryS}} \theta_{c\mu} \phi_{c0} \exp \left(-\frac{\theta_c}{K_{\text{dryS}}} P_{\text{eff}} \right) \quad (3.56)$$

$$V_{\text{Pdry}}(P_{\text{eff}}) \approx V_{\text{PdryS}} + \frac{1}{2} V_{\text{SdryS}} \theta_s H_s \left(\frac{1}{K_{\text{dryS}}} - \frac{1}{K_0} \right) P_{\text{eff}} - \frac{1}{2} V_{\text{PdryS}} \theta_c H_c \phi_{c0} \exp \left(-\frac{\theta_c}{K_{\text{dryS}}} P_{\text{eff}} \right), \quad (3.57)$$

with

$$V_{\text{SdryS}} = \sqrt{\frac{\mu_{\text{dryS}}}{\rho}}, \text{ and } V_{\text{PdryS}} = \sqrt{\frac{K_{\text{dryS}} + \frac{4}{3} \mu_{\text{dryS}}}{\rho}},$$

and

$$H_s = \frac{\frac{K_{\text{dryS}} \theta_s}{\theta_{s\mu}} + \frac{4\mu_{\text{dryS}}}{3}}{K_{\text{dryS}} + \frac{4\mu_{\text{dryS}}}{3}}, \quad H_c = \frac{\frac{K_{\text{dryS}} \theta_c}{\theta_{c\mu}} + \frac{4\mu_{\text{dryS}}}{3}}{K_{\text{dryS}} + \frac{4\mu_{\text{dryS}}}{3}}$$

If these approximations are sufficient the stress dependence of the velocities has also the form of equation 1.1.

In order to clarify the error introduced by fitting the linear approximations (3.56) and (3.57) in comparison to the exact relations (3.53) and (3.54) the dry sandstone sample 131 from Freund (1992) is considered. Figure (3.5(a)) shows the stress dependent P-wave velocity (red dots). The velocity increased from 3.18 km/s at 8 MPa to 5.11 km/s at 300 MPa effective stress, hence, by 60%. The linear approximation solution was obtained by fitting the observed P- and S-wave velocities with equation (1.1). Assuming negligible density variations with stress K_{dry} and μ_{dry} were calculated from the velocity data and fitted with equations (3.51) and (3.52). The obtained best fit parameters for K_{dry} and μ_{dry} were then used to calculate the stress dependent P-wave velocity with equation (3.54). The resulting P-wave velocity is shown as dashed blue

3.4. Stress sensitivity of electrical resistivity

line in Fig.(3.5(a)). The dashed green line shows the P-wave velocity obtained from the best fit parameters obtained from applying the linear approximation to the data. Obviously, both velocities fit the observed velocities with a very high accuracy. A visible deviation of the approximation from the exact solution appears for $P \rightarrow 0$ only. Figure (3.5(b)) shows the relative deviation V_{Prd} of the approximation $V_{Papprox}$ from the exact solution V_{Pexact} , calculated as

$$V_{Prd} = \frac{V_{Pexact} - V_{Papprox}}{V_{Papprox}} \quad (3.58)$$

At zero effective stress the error due to the application of the linear approximation in contrast to the exact equation is 8% and decreases rapidly. At 8 MPa (first observation stress) the deviation is 0.8% only. Taking into account that Freund (1992) gives an error in measurements of 2% and 3% for P- and S-wave, respectively, the error due to the application of the linear approximation seems to be negligible, although this sample shows a strong increase of velocities with stress.

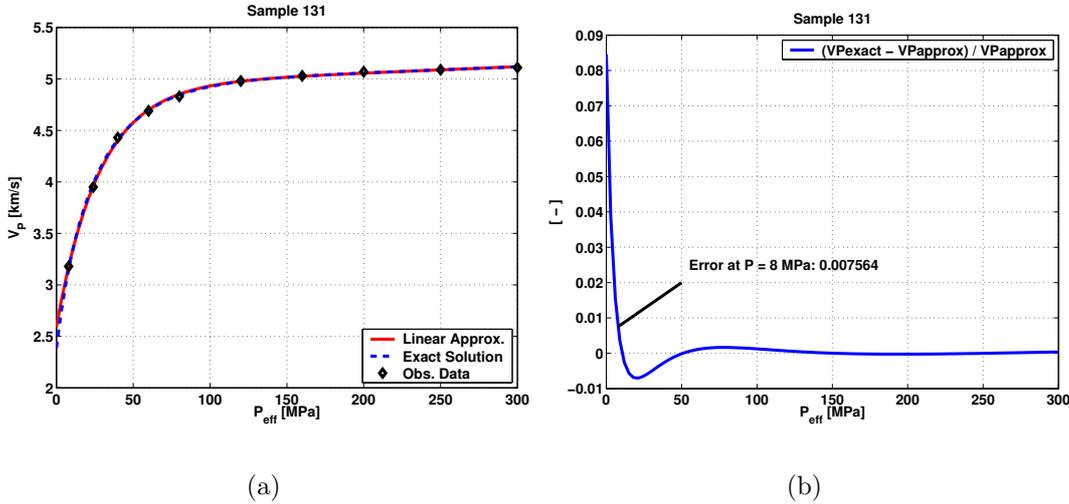


Figure 3.5: Fig. (3.5(a)): Exact stress dependent P-wave velocity (green dashed line) and linear approximation (blue dashed line) fit observed P-wave velocities (red dots) quite well. Fig. (3.5(b)): The maximum error introduced due to the application of the approximation is 8% at zero confining stress. At the first observed velocity the deviation has already decreased to 0.8%.

3.4 Stress sensitivity of electrical resistivity

In the previous part of this thesis it was shown that the stress dependence of various elastic characteristics of porous rocks might be understood as a result of pore space deformation. If this approach is valid it should also be possible to extend it to other rock properties, as long as they depend somehow linearly on the porosity. Especially the transport properties of rocks, i.e., electrical resistivity and hydraulic conductivity (or likewise permeability) are potentially important candidates for such considerations.

However, the dependence of the transport properties on porosity is highly complicated and not straightforward, since they do not depend on the amount and geometry of the pore space alone, but also on its connectivity. In fact, there are numerous examples in the literature where permeability and porosity are positively correlated and not less examples for which a negative correlation is reported. Some approaches exist to describe the influence of stress on permeability where fluid flow is assumed to be controlled by fractures, (e.g., Gangi, 1978; Tsang & Witherspoon, 1981; Raven & Gale, 1985). However, these approaches are usually based on specific assumptions about the geometry of the fractures in terms of the cubic law. Specific considerations about the geometry of the fractures do not correspond to the more general treatment of cracks and fractures as considered in the stress sensitivity approach. Thus, permeability will not be considered in the following.

Electrical resistivity and porosity are also related to each other in a quite complex manner. Beside the influence of the connectivity of the pore space on the rocks resistivity, a reasonable and applicable analysis of the porosity dependence may be additionally impeded by clay minerals and highly conductive ore phases. However, Archie (1942) found a quite simple empirical resistivity-porosity relation for rocks, where only electrolytic charge transport through an interconnected pore space occurs. This relation will be considered in the next section.

It is well known that electrical resistivity is remarkably more sensitive to porosity, temperature, and fluid saturation than seismic velocities (e.g., Wilt & Alumbaugh, 1998). Numerous laboratory experiments have been conducted in the past to understand the electrical properties of very different rock types. A review of these studies can be found, e.g., in Wyllie (1963); Olhoeft (1980); Parkhomenko (1982). However, in high porosity reservoir rocks electrical resistivity is usually assumed to be independent from changes of the in situ stress field. For example, Daily & Lin (1985) found that the electrical conductivity primarily resulted from electrical volume conduction and that resistivity is not affected by changing elastic moduli through crack closure due to compression as long as the large aspect ratio pores remain open. Lockner & Byerlee (1985) compared the stress dependent complex resistivity of Westerly granite with the one of Berea sandstone. They found that the stress dependence of electrical resistivity is much smaller in sandstones than in granites. For one granitic sample the real part of the low-frequency conductivity dropped of by 94 % at 200 MPa confining isostatic stress whereas the conductivity of the sandstone decreased by only 24 %. Crystalline rocks seem to behave like sandstones when partially saturated (Brace & Orange, 1968).

This section attempts to explain observations referred to above. It follows the concept of stress sensitivity, introduced before. However, the dependence of electrical resistivity and hydraulic on porosity is more sophisticated. In order to obtain a stress dependence of electrical resistivity using the load dependent deformation of the pore space, we have to restrict the derivations to rocks where only electrolytic charge transport through an interconnected pore space occurs.

Formulating the stress dependence of electrical resistivity in terms of stress sensitivity as presented for elastic moduli and compliances requires the limitation to rocks where only electrolytic charge transport is assumed to take place, i.e., surface conductivity should be negligible and highly conducting mineral phases should not be present. In other words, we restrict our considerations to rocks where electrical resistivity can

3.4. Stress sensitivity of electrical resistivity

be described by the well known Archie's law (Archie, 1942).

$$\frac{1}{\Omega} = \frac{1}{F\Omega_{fl}} = \frac{1}{\Omega_{fl}}\phi^m. \quad (3.59)$$

Here, Ω_{fl} is the resistivity of the pore fluid, F is the formation factor, ϕ is the porosity, and m is Archie's cementation exponent. In general, m is in the range $1 \leq m \leq 2$ but occasionally reaches 2.3 (Berryman, 1992). Archie's law shows that electrical resistivity is in general not a linear function of porosity. In fact, only in the special case $m = 1$ electrical resistivity linearly depends on porosity. However, equation (3.59) shows that the logarithm of resistivity depends linearly on porosity.

Rearranging equation (3.59), using again $\phi = \phi_{s0} + \phi_s + \phi_c$ and taking the logarithm gives:

$$\log \frac{\Omega}{\Omega_{fl}} = -m \cdot \log \phi = -m \cdot \log(\phi_{s0} + \phi_s + \phi_c). \quad (3.60)$$

Obviously, the logarithm of the formation factor $F = \Omega/\Omega_{fl}$ is a linear function of porosity.

Using a Taylor expansion gives:

$$\log \frac{\Omega}{\Omega_{fl}} = -m \log \phi_{s0} - \frac{m}{\phi_{s0}}\phi_s - \frac{m}{\phi_{s0}}\phi_c. \quad (3.61)$$

Now using the stress dependent formulations for ϕ_s and ϕ_c as given by Shapiro (2003), finally gives:

$$\log \frac{\Omega}{\Omega_{fl}} = -m \log \phi_{s0} - \frac{m}{\phi_{s0}} \left(\frac{1}{K_{dryS}} - \frac{1}{K_0} \right) P - \frac{m}{\phi_{s0}} \phi_{c0} \exp \left(-\frac{\theta_c}{K_{dryS}} P \right) \quad (3.62)$$

Comparing equation (3.62) with (1.1) illustrates the physical meaning of the fit parameters A , K , B , and D in the case of stress dependence of logarithmic formation factor:

$$A = -m \log \phi_{s0}, \quad (3.63)$$

$$K = -\frac{m}{\phi_{s0}} \left(\frac{1}{K_{dryS}} - \frac{1}{K_0} \right), \quad (3.64)$$

$$B = \frac{m}{\phi_{s0}} \phi_{c0}, \quad (3.65)$$

$$D = \frac{\theta_c}{K_{dryS}}. \quad (3.66)$$

Fit parameter A corresponds exactly to Archie's law if ϕ in equation (3.59) is equal to the stress independent part ϕ_{s0} of the bulk porosity. In the case of fit parameter D we obtain the same expression as for elastic moduli and seismic velocities.

Here, however, the fit parameters K and B are significantly different in comparison to their corresponding formulation in the case of elastic moduli and velocities. The magnitudes of K and B are proportional to $1/\phi_s$ and ϕ_c/ϕ_{s0} , respectively, while they are proportional to $\theta_c\phi_s$ and $\theta_c\phi_c$ in the case of the other elastic moduli and velocities. This has an important consequence. In most reservoir rocks stiff porosity and even the

stress induced change in stiff porosity is much larger than compliant porosity. Despite this fact the closure of the crack porosity is dominant for the stress dependence of the elastic moduli and velocities. These properties are rather sensitive to the relative changes of the different porosity domains, expressed in the θ terms ($|\theta_s| \ll |\theta_c|$), than to the absolute changes (Shapiro, 2003).

In contrast, equation (3.62) states that the stress dependence of electrical resistivity is controlled by the absolute change of the porosities. Consequently, the change in stiff porosity controls electrical resistivity as a function of effective stress in reservoir rocks and not the change in compliant porosity. In turn, the stress dependence of stiff porosity can be neglected over the effective stress range of interest (up to 200 MPa). This might be the reason that electrical resistivity is usually assumed to be independent from stress.

3.5 Stress sensitivity of Poisson's ratio

From many laboratory experiments it is known that Poisson's ratio ν or likewise the V_p/V_s -ratio, is one of the rock physical parameters most sensitive to lithology and to the type of rock saturating fluid. Domenico (1984) examined sandstone, limestone and dolomite P- and S-wave data. He found that these rock types are clearly separated by ν (see Tab. 3.1) and addresses the separation of sandstones and limestones to the distinct Poisson's ratio of the dominant matrix minerals, namely quartz ($\nu = 0.056$) and calcite ($\nu = 0.316$).

Table 3.1: Typical Poisson's ratio for sandstone, limestone, and dolomite (Domenico, 1984)

Rock type	Poisson's ratio
Sandstone	0.17–0.26
Limestone	0.27–0.29
Dolomite	0.29–0.33

Carcione & Cavallini (2002) suggest the existence of a precise relation between Poisson's ratio on the one side and pore pressure and fluid type on the other side for isotropic and anisotropic rocks. They found that ν is approximately constant at high differential pressure and decreases for dry rocks at low differential pressure. In contrast, they found an increasing ν at low differential pressure when the rocks are saturated. Moreover, in samples of equal porosity they found that ν depends on the aspect ratio of the cracks and pores and the saturating fluid. Rocks consisting mainly of stiff porosity show a less sensitivity of ν upon effective stress. Thus, it is consequent to investigate ν in terms of the stress sensitivity approach.

Assuming isotropic rocks Poisson's ratio can be calculated from the elastic moduli and seismic velocities. The corresponding relations read:

$$\nu = \frac{1}{2} \left(1 - \frac{1}{a-1} \right), \text{ with } a \equiv \left(\frac{V_p}{V_s} \right)^2 \quad (3.67)$$

3.5. Stress sensitivity of Poisson's ratio

and

$$\nu = \frac{3K - 2\mu}{2(3K + \mu)} \quad (3.68)$$

Assuming a material with very high rigidity ($\mu, V_s \rightarrow \infty$) or a fluid ($\mu, V_s \rightarrow 0$) shows that $-1 < \nu < 0.5$.

In order to define the stress dependence of ν in terms of terms of the stress sensitivity approach it is straightforward to insert the corresponding equations for moduli and velocities into equations (3.67) and (3.68). This gives for eq. (3.67):

$$\nu = \frac{1}{2} \left(1 - \frac{1}{\left(\frac{A_P + K_P P_{\text{eff}} - B_P \exp(-D_P P_{\text{eff}})}{A_S + K_S P_{\text{eff}} - B_S \exp(-D_S P_{\text{eff}})} \right)^2 - 1} \right), \quad (3.69)$$

and

$$\nu = \frac{3A_K - 2A_\mu + (3K_K - 2K_\mu)P_{\text{eff}} - 3B_K \exp(-D_K P_{\text{eff}}) - 2B_\mu \exp(-D_\mu P_{\text{eff}})}{2[3A_K + A_\mu + (3K_K + K_\mu)P_{\text{eff}} - 3B_K \exp(-D_K P_{\text{eff}}) - B_\mu \exp(-D_\mu P_{\text{eff}})]} \quad (3.70)$$

for eq. (3.68). Considering the physical characteristics defining the fit parameters used in eq. (3.70) and assuming that $D = D_K = D_\mu$ gives:

$$\nu = \frac{3K_{\text{dryS}} - 2\mu_{\text{dryS}} + (3\theta_s - 2\theta_{s\mu}) \left(\frac{1}{K_{\text{dryS}}} - \frac{1}{K_0} \right) P_{\text{eff}} - (3\theta_c + 2\theta_{c\mu}) \phi_{c0} \exp(-\theta_c / K_{\text{dryS}} P_{\text{eff}})}{2 \left[3K_{\text{dryS}} + \mu_{\text{dryS}} + (3\theta_s + \theta_{s\mu}) \left(\frac{1}{K_{\text{dryS}}} - \frac{1}{K_0} \right) P_{\text{eff}} - (3\theta_c + \theta_{c\mu}) \phi_{c0} \exp(-\theta_c / K_{\text{dryS}} P_{\text{eff}}) \right]} \quad (3.71)$$

In the same way eq. (3.69) can be expanded towards the physical rock characteristics behind the used fit parameters and simplified by assuming $D_P = D_S$.